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RESEARCH ARTICLE

Computational Approach to Alternative Goldbach Conjecture

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Abstract

In this article, we will review some historical attempts to prove Goldbach Conjectures. A few flaws presented in some weak induction which used in those attempting to prove. A few examples presented to demonstrate the Alternative to Goldbach Conjecture (A.G.C). A computer program developed with C^{++} to generate the Goldbach Pair of Primes.

Keywords: Mersenne Prime; Fermat Prime; Goldbach"s Conjecture (G.C); Alternative to the (G.C); Conjugate Pair of Primes

Introduction

In a 1742 letter to Leonard Euler, the German mathematician Cristian Goldbach made the conjecture that- " Every even integer greater than 2 can be written as the sum of two primes ", Neither Euler or anyone else since that time has been able to prove this conjecture although computer tests have shown it to be correct through primes of at least 18 digit length.

In searching prime numbers, Father Marin Mersenne (1588-1648) was interested in integers of the form $2^n - 1$. He showed that these numbers could only be prime if n is prime. Then he discovered that it wasn't sufficient for just n to be prime since he found a counter example

$$2^{11} - 1 = 2047 = 23 * 89$$

That is not a prime number. He conjectured that $2^p - 1$ is a prime number when p is prime but unfortunately he was wrong. In Mersenne honor, the set of prime numbers generated by this expression is called Mersenne primes.

This statement that he made: *"To decide, whether a number of 15- or 20-digits is prime or not, it is not enough even a lifetime, no matter how we use all of our knowledge" – Marin Mersenne, 1644.*

In 2001, 20 year old Michael Cameron discovered the largest value for p=13466917 produced a prime 2^p-1 equivalent to 4,053,946 digits after two and a half day testing 100,000 sample numbers.

Fermat also produced a different expression $2^{2^n} + 1$ to generate prime factors called Fermat primes.

Goldbach Conjecture (GC) dates backs to the letter from Christian Goldbach to Leonhard Euler on June 7, 1742. Many mathematicians have tried to provide a proof for this conjecture during the last couple of centuries. Some approaches proposed for this purpose can be found in mathematic papers references but the strong Goldbach conjecture is still known as an open problem.

Goldbach's conjecture has been one of best-known unsolved problems in mathematics for a long time, which was listed as a part of Hilbert's 8th problem in 1900 [5]. The conjecture includes strong and weak statements. The strong Goldbach conjecture states that every even number greater than 2 is the sum of two primes, which is equivalent to the statement that every even number greater than 4 is the sum of two odd primes.

The weak Goldbach Conjecture states that every odd number greater than 7 is the sum of three odd primes.

Many progresses have been made in the last few decades. In particular, in 1997 it was shown that the Goldbach conjecture is related to the generalized Riemann hypothesis in the sense that Riemann implies the Goldbach weak conjecture for all numbers [1].

Historical Attempts to Prove Goldbach Conjecture (GC)

For three centuries many attempts have been made to present the proof of Goldbach's conjecture. Some tried to present a heuristic proof, numerical approach, or claimed a rigorous proof. In reference [1] Miles Mathis presented a heuristic proof: (see milesmathis.com/gold3.html) using the density of pair of primes in the decomposition method in a Goldbach set of integers.

There is serious doubt in the rigorous proof claimed in reference [2] about the Goldbach conjecture.

They write an integer N in addition of two integers by adding and subtracting some other even integer: $N=2n+6=(3+2i)+(2n+3-2i)$, for some positive integers $i=0,1,2,3,\ldots$They tried to demonstrate or verify that there exists some integers i which will make any even integer N as the sum of two primes. They called their approach a rigorous proof for Goldbach's Conjecture. It is a weak induction which it totally wrong to say that it is a rigorous proof. It is merely exploring to verify that the conjecture is true.

In references [3] Romanov and [4] William West some proofs by induction were presented. In this paper we try to explore the alternative to Goldbach's conjecture by a computational Algorithm [3,4].

Goldbach's conjecture is one of the oldest unsolved problems in number theory and in all of mathematics. Based on Strong Goldbach Conjecture (G.C) *every even integer N* greater than two is the sum of two primes. It is important for any proof to demonstrate that GC is true for all even integers.

Example (1): As a beginner we may search two prime numbers which is equal to the given even integer: $4= 2+2$, $8= 5+3$, $12= 5+7$, ...

The ternary Goldbach conjecture (or three-prime problem) states that every odd number N greater than 7 can be written as the sum of three primes (notice that integer 1 is also considered as a prime number).

Example (2): Observe that $7=3+2+2$, and $19=7+7+5,\ldots$

You may experiment by exploring other odd numbers as a sum of three primes or develop a computer program to generate the desired prime numbers.

Computational Approach an Alternative method to verify the Goldbach Conjecture (A.G.C)

Since all other approaches to the proof of the Goldbach conjecture is either not true or unclear,

We will follow a different approach that may help more and be useful to the final conclusion.

Statement of the problem: For every natural number n greater than or equal to "4", there is at least another natural number (k) such that "$n+k$" and "$n-k$" are prime numbers.

Alternative to the Goldbach Conjecture (A.G.C): Assume that Z is the set of all integers and $N \in Z^+$. For every even natural number $N = 2n$, where $n \in Z^+$ **there** exists a positive integer k such that both are prime numbers.

$$n-k \;\; and \;\; n+k \tag{1}$$

Let us explore this statement by presenting a few examples.

Example (3): Assume that an *even integer N=30* demonstrates the Alternative Goldbach Conjecture for this integer. Notice that the even number N=2n=30, thus n=15. We may use spread sheet excel to write $n=1,\ldots,15$ in the first row and another row $n=15,\ldots,29$ reverse order in the second row. We may observe that for $n=\frac{N}{2}$ there exists some Positive integer k such that $n-k$ and $n+k$ are prime numbers.

For instance if k=2 then there are a pair $(15-2, 15+2) = (13,17)$are prime and $13+17=30$. We can find other pairs of prime, for $k=4, 8$. It will not be true for k= 14 since 1 is not considered as a prime number.

The sum of these pair of primes p and q which is equal to an even number and is denoted here by "e" are called conjugate primes where p+q=e. This example, like others that we will explore, demonstrates that the distribution of the pair of primes is equidistant from the center of each even positive integer. The set of all prime pairs can be described by:

$$<p, q> = \{(13,17), (11,19), (7,23)\}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
29	28	27	26	25	24	23	22	21	20	19	18	17	16	15

Table 1: This table demonstrates the set of all conjugate prime numbers satisfies the Goldbach Conjecture

Example 4: Repeat the method of Exercise 3 for even integer N=2n=40. a) Show that the conjugate of pair of primes is in the following form:

$$<p, q> = \{(17,23),(11,29),(3,37)\} \tag{2}$$

b) Determine the values of integer k defined in (1).

Properties of Goldbach Conjugates Pair of Primes (G.C.P.P)

Using the previous examples and the relation (1), it is reasonable to define a set of Golbach pair of prime numbers as follows:

$$< p,q > \;=\; \{(p,q) \in N \times N : p \,\&\, q \text{ are prime and } p+q = N = 2n\} \tag{3}$$

Assume that the A.G.C is true then a positive integer k exists in (1). Rewriting p=n-k and q=n+k implies that <n-k,n+k> will be a G.P.P.

Another conclusion of the relations (1) to (3) is very obvious that the G.P.P are distributed in the set {1, 2, 3...n, n+1,...N} equal distant from the center n.

Assume that the A.G.C is true for some pair of primes (p_j, q_j) for $j= 1,2,3, \dots l$. The integer l represents the number of G.P.P and now we can assume it will be between 0 and n. If l=0 for some even integer N=2n, that means the G.C fails. Thus, to study the Goldbach pair of primes we assume $0 <l<n$.

Lemma (1): The Goldbach Conjugate Pair of Primes (G.C.P.P) (p_j, q_j) are distributed equidistant from the center (n) *for* $j = 1,2,3, \dots l$ that is

$$n - p_j = q_j - n \text{ for all } j \;=\; 1,2,3,\dots,l$$

By a little algebraic manupulation,the proof is immediatesince

$$p_j + q_j = 2n = N \text{ for all } j \;=\; 1,2,3,\dots,l \tag{4}$$

Notice that in Table 1 of Example (3), we have N=30 and n=15. The distance of a pair of prime (13,17) from the center is: 15-13 =2 and 17-15=2.

Lemma (2): There exist constant integers a and b (a and b in Z) such that for all
Conjugate Pairs of Primes p and q, $a.p + b.q = 1$.

Brief Proof: The conjugate pair of primes will also be relatively prime and their greatest common divisor is equal to (p,q)=1. Thus, there exist integers a and b such that a linear relation is true.

A graph of this linear relation can be observed in Figures 3 and 4.

Partitioning the set of Ordered Golbach's Conjugate Pairs of Primes in $N\times N$ of the Cartesian plane: Mile Mathis presented "The Simple Proof of Goldbach's Conjecture" using the interesting partitioning approach. Let us call the set of all even numbers E a subset of a given number N and denote P the set of all prime numbers in N.

It is understood that the intersection of $E \cap P = \{2\}$. This shows that 2 is the only even prime number. Odd numbers in N also can be partitioned by two subsets of "odd primes" and "non-primes odd". Just for example

$$< p,q > \;=\; \{(p,q) \in N \times N : p \,\&\, q \text{ are prime and } p+q = N = 2n\} \tag{5}$$

Many researchers made partitions based on two characteristic categories of numbers {P,C} which represent a set {Prime, Composite}. To find the distribution of conjugate primes we will have the following set in the in the Cartesian plane of $N \times N$.

$$N \times N = \{(C,C),(C,P),(P,C),(P,P)\} \tag{6-a}$$

It is our obligation to demonstrate that the partition (6) is a weak demonstation to describe all relations in the $n \times n$ set of Cartesian product. Let us demonstrate a table for Goldbach Conjecture Pair of Primes (G.C.P.P) similar to the TABLE (1) for $n \times n$ as subset of $N \times N$. In the following examples we will show the difference between the cardinality of both sets N and nxn.

Assume that N=2n and E is the set of all even integers in N. It is clear that

$$|E| = \frac{N}{2} = n$$

Let us denote p the set of all odd prime P and p' the number of odd non-prime P' including unit {1}. Thus the set of all even positive integers N can be described as the sum of the number of even integers n, the number of odd primes, the number of odd non-prime, excluding elemnt {2}

$$N = n + p + p' - 1. \tag{6-b}$$

The relation (6-b) represents simple linear relations between the elements in the set N. In Example (3), the relation (6-b) will be:

$$|N| = n + p + p' - 1 = 15 + 10 + 6 - 1 = 30$$

Example (5): Table 4 represents a demonstration of Goldbach Conjecture when N=60 is selected. Thus, we have $|N| = 60$, $n = 30$, $|P| = 17$, |p'|=14, number of odd-prime=6, and nonprime odd=14. Then $N = n + p + p' - 1 = 30 + 17 + 14 - 1 = 60$.

Notice that the element 2 is the only even prime number and it is a common element in the two sets E and P.

General Partition Method for Altenative Goldbach Conjecture: As a result the set nxn will be described by

$$n \times n = \{(e,e),(p,p),(p,p'),(p,e),(p',e),(p',p),(p',p')\} \tag{7}$$

Thus we have the cardinality

$$|n \times n| =$$

$$|<e,e>| + <p,p> | + |<p,p'>| + |<p,e>| + |<p',e>| + |<p',p>| + | <p',p'>| \tag{8}$$

We can verify a new partition approach (8) for N=60 and demonstrate the numerical values of the sets introduced above.

$$|<e,e>| = 6, <p,p>| = 3, |<p,p'>| = 2, |<p,e>| = 1, |<p',e>| = 0,$$

$$|<p',p>| = 1, |<p',p'>| = 2,$$

$$|n \times n| = 6 + 3 + 2 + 1 + 0 + 1 + 2 = 15$$

Example (6): We will use one more practical exercise borrowed from the reference [9]. This is to demonstrate a set of positive integers with size N=2n=62. Thus for n=31 the cardinality of elements in the set nxn will be the following:

$$|<e,e>| = 15, |<p,p>| = 3, |<p,e>| = |<e,p>| = 0, |<p,p'>| = 7, |<p',p>| = 4, |<p',p'>| = 2$$

We plug in the relation (5) to verify the new partition approach:

31=15+3+0+0+7+4+2=n.

These cardinalities obtained manually through the Table 5. A mathematical closed form of production for these cardinalities is an open problem.

Computation Approach to Alternative Goldbach Conjecture

Alternative to Goldbach Conjecture

There are primes: n-k and n+k

n	k	n-k	n+k
4	1	3	5
5	2	3	7
6	1	5	7
7	4	3	11
8	3	5	11
9	2	7	11
10	3	7	13
11	6	5	17
12	1	11	13
13	6	7	19
14	9	5	23
15	2	13	17
16	3	13	19
17	6	11	23
18	11	7	29
19	12	7	31
20	17	3	37

Table 2: Alternative to G.C for small size of N=20

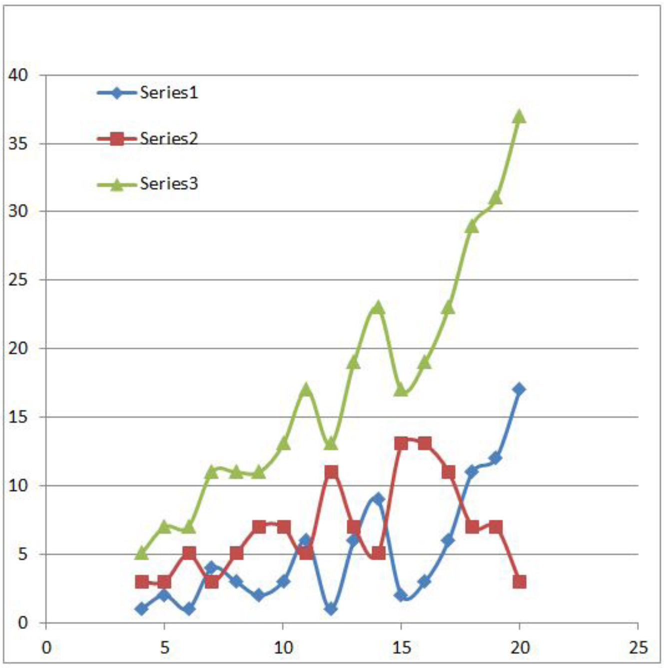
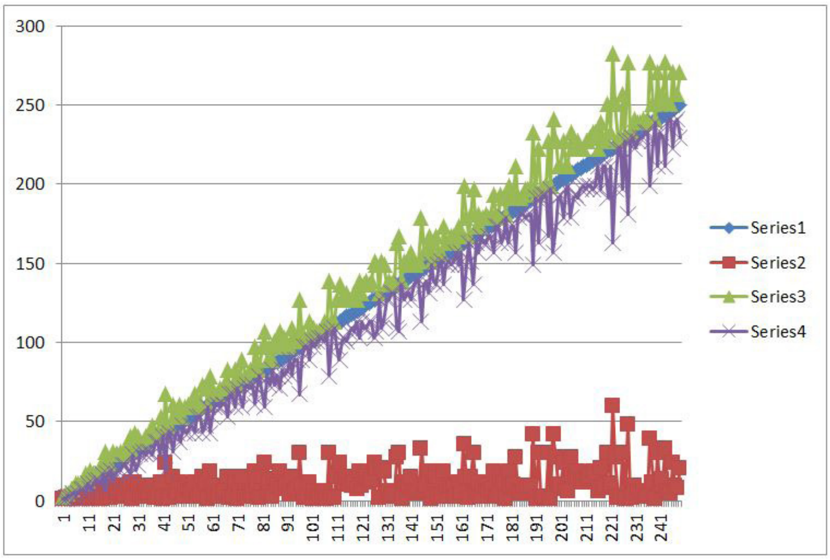


Figure 1: Existing of the Goldbach Prime for small size of a set with N=20

A computational algorithm is presented in the appendix of this paper showing a different approach to verify and demonstrate the Alternative to Goldbach's Conjcture (AGC). This is not a proof for existence of the pair of primes. To prove this conjecture one should demonstrate that the set $<p, q> \neq \emptyset$ is non-empty. The pair of prime conjugates may be easy to demonstrate manually. In the previous tables for small even positive integers like N=30, 40, 60, or 62 the table set for $<p,q>$ are presented manually (Graph 2).



Graph 2: Distribution of the Goldbach pair of primes produces by the program in Appendix

A Note for the proof of the Strong Goldbach Conjecture

It states that every even integer greater than 4 can be written as the sum of two primes. As a result, equivalently, one may claim that given the Goldbach Conjecture is proved, then for every even integer N=2n there exists a positive integer k

$$N = \frac{N}{2} + \frac{N}{2} = (n+k)+(n-k) \tag{9}$$

Such that p=n-k and q=n+k will be prime numbers.

This assertion demonstrates that the Goldbach Conjecture implies the Alterative Conjecture.

Conversely; assume that the Alternative G.C is true. That is for every positive even integer N=2n, there exists another integer k such that (9) holds for two prime numbers p=n-k and q=n+k. This is the immediate result that shows the A.G.C is true.

In our numerical and graphical approach, we cannot claim this demonstration is a proof for G.C. In proving the A.G.C, we need to prove the existence of positive integer k and also prove that p and q are prime numbers.

Computational or Graphical Approaches to the Alternative Goldbach Conjecture

All of the historical attempts are fascinating numerical, graphical, and analytical approaches to demonstrate and verify the GC properties.

Markakis 2013 and Kurzweg 2016 are using a numerical distribution of primes and also tried to show the link of this model to pair of primes [6,8].

Both of these authors used congruence class of modulo (6) that can be described by all of the congruence classes of remainders {0,1,2,3,4,5}. All of these two classes can be described by

$6m \pm r$ for any positive integer m. Since remainders 0, 2,4 and 3 will not generate prime numbers we can include all prime numbers in two sets $6m \pm 1$ for remainders 1 and 5. These two sets contain all prime numbers.

It is an excellent educational exploration in reference [8], but we may see the following statement in reference [6] that we may observe Kurzweg 2016 [6]. "Goldbach Pairs and Goldbach's Conjecture) conclude [6,8]:

"What is quite clear from these results is that GP increases as a power of N, meaning that for $N \geq 12$ for which the $N = 6n \pm 1$ forms apply, will have at least one Goldbach Pair available to confirm Goldbach's Conjecture. When this fact is taken into account together with the list given at the beginning of this article, it is clear that the Goldbach Conjecture is true for all even numbers N."

It is important to notice that, after using a weak induction, no matter how many billion times we experiment, one cannot conclude that the statement for G.C is true for all integers.

To verify the Alternative to Goldbach's Conjecture, we first demonstrated manually with small even positive integers how to find the conjugate pair of primes.

With the computational approach, a program was developed for any arbitrary even integer $\in Z^+$.

Conclusion

It is very important to notice that all of the fascinating explorations of the Goldbach Conjecture will not be a substitute for a mathematical proof based on strong mathematical induction that can demonstrate that the result is true for all $N \in Z^+$. One of the main reasons is that we can observe that they are missing the Power of the notion on the universal quantifier "for all n", "for every n", or "$\forall n \in Z^+$

In mathematical induction to show a statement S (n) is true for all , we show that S (1) is true and also when S (n) is true then S (n+1) is true.

Theoretically, when you conclude any logical statement based on some list of the axioms after trying an experiment n-times with the same result, it will not guarantee the same result for the (n+1)- experiment.

That means we are using a *weak induction* that cannot be generalized for all positive integer n.

Although all of these explorations with numerical or graphical approaches have their own educational values, and they are excellent ways for inspiring motivation for getting close to the final proof but is not considered a mathematical proof.

In this paper we presented an alternative verification for Goldbach Conjecture using C++ programming.

Appendix (1)

```
#include <iostream>
#include <fstream>
```

```
using namespace std;
// prime function: it is for checking n-c, n+c that are prime or not
bool prime(int num)
{

int    i,  flag =0;

for(i = 2;  i <= num/2;  ++i)
{
if(num % i == 0)
{
      flag = 1;
      break;
     }
}

if  (flag == 1 )
{
    return false;
}
else
{
      return true;
    }
}

//get_constant: get constants c and prime pairs for each number (n)
bool get_constant(int n){
   int sign=0;
   int temp=1;

   int counter=0;
   int m=n;
if (n%2==0)  {n=n/2;
                              temp=0;}

ofstream MyExcelFile, MyExcelFile2;
MyExcelFile. open("Home: \\test2 . csv", ios: :app);
MyExcelFile2. open("Home: \\number  of  primes . csv",ios: :app);
if(MyExcelFile.fail())

MyExcelFile  <<  "number, temp, n+temp, n-temp"  <<endl;

While (temp<n) { if   (prime(n+temp)  &&  prime(n-temp))
                    {sign++;
                    if (sign==1)
                    {cout<<m<<"       "<<n+temp<<"     "<<n-temp<<" ";
                    MyExcelFile  <<m<<" ,"<<n+temp<<" ,"<<n-temp<<" ,";
                    counter++;
                    }else
                    {cout<<""<<temp<<"         "<<n+temp<<"     "<<n-temp<<" ";
                    MyExcelFile<<" ,"<<n+temp<<" ,"<<n-temp<<" ,";
                    counter++;
                    }
                    temp++;
                                }
                    else
                         temp++;
                    }
```

```
                    cout<<"\n";
                    MyExcelFile<<endl;
                    MyExcelFile2<<m<<" ,"<<"number of pairs"<<"

,"<<counter<<endl;
                      MyExcelFile.close();
                      MyExcelFile2.close();
                      if (temp>n) {cout<<"for"<<n<<"not found";
                                  return false;}
                                  else return true;
}
int main(int args, char * argv[])
{
    for (int n=2; n<=100; n+=2)
    {
        if(get_constant(n)==false) break;
    }
    return 0;
}
```

Program Output to Show all Altenative Goldbach Pair of Primes (A.G.P.P)

n=2n	p	q																#G.P.P
2																		0
4	2	2																1
6	3	3																1
8	5	3																1
10	5	5	7	3														2
12	7	5																1
14	7	7	11	3														2
16	11	5	13	3														2
18	11	7	13	5														2
20	13	7	17	3														2
22	11	11	17	5	19	3												3
24	13	11	17	7	19	5												3
26	13	13	19	7	23	3												3
28	17	11	23	5														2
30	17	13	19	11	23	7												3
32	19	13	29	3														2
34	17	17	23	11	29	5	31	3										3
36	19	17	23	13	29	7	31	5										3
38	19	19	31	7														2
40	23	17	29	11	37	3												3
42	23	19	29	13	31	11	37	5										4
44	31	13	37	7	41	3												3
46	23	23	29	17	41	5	43	3										4
48	29	19	31	17	37	11	41	7	43	5								5
50	31	19	37	13	43	7	47	3										4
52	29	23	41	11	47	5												3
54	31	23	37	17	41	13	43	11	47	7								5
56	37	19	43	13	53	3												3
58	29	29	41	17	47	11	53	5										3
60	31	29	37	23	41	19	43	17	47	13	53	7						6
62	31	31	43	19	59	3												3

n=2n	p	q																#G.P.P
64	41	23	47	17	53	11	59	5	61	3								5
66	37	29	43	23	47	19	53	13	59	7	61	5						2
68	37	31	61	7														2
70	41	29	47	23	53	17	59	11	67	3								5
72	41	31	43	29	53	19	59	13	61	11	67	5						6
74	37	37	43	31	61	13	67	7	71	3								5
76	47	29	53	23	59	17	71	5	73	3								5
78	41	37	47	31	59	19	61	17	67	11	71	7	73	5				7
80	43	37	61	19	67	13	73	7										4
82	41	41	53	29	59	23	71	11	79	3								5
84	43	41	47	37	53	31	61	23	67	17	71	13	73	11	79	5		8
86	43	43	67	19	73	13	79	7	83	3								5
88	47	41	59	29	71	17	83	5										4
90	47	43	53	37	59	31	61	29	67	23	71	19	73	17	79	11	8	9
92	61	31	73	19	79	13	89	3										4
94	47	47	53	41	71	23	83	11	89	5								5
96	53	43	59	37	67	29	73	23	79	17	83	13	89	7				7
98	61	37	67	31	79	19												3
100	53	47	59	41	71	29	83	17	89	11	97	3						6

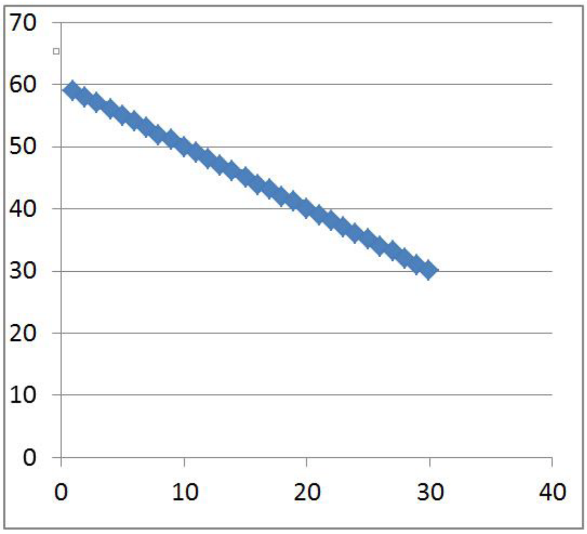
Table 3: This table is the result of a computer program which demonstrates the Alternate Goldbach Pair of Primes for every even integer N=2n there exists a positive integer k such that both p=n-k and q=n+k are prime numbers

Appendix (2)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30

p	7	13	17	19	23
q	53	47	43	41	37

Table 4: Goldbach Conjecture and Conjugates Pair of Primes for N=60



(a)

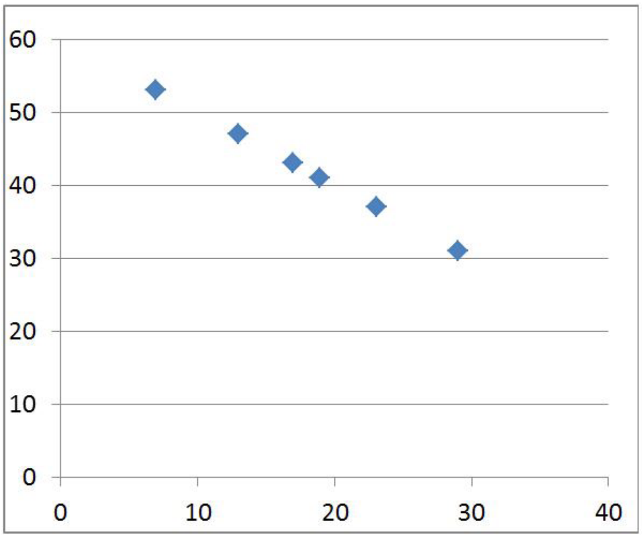
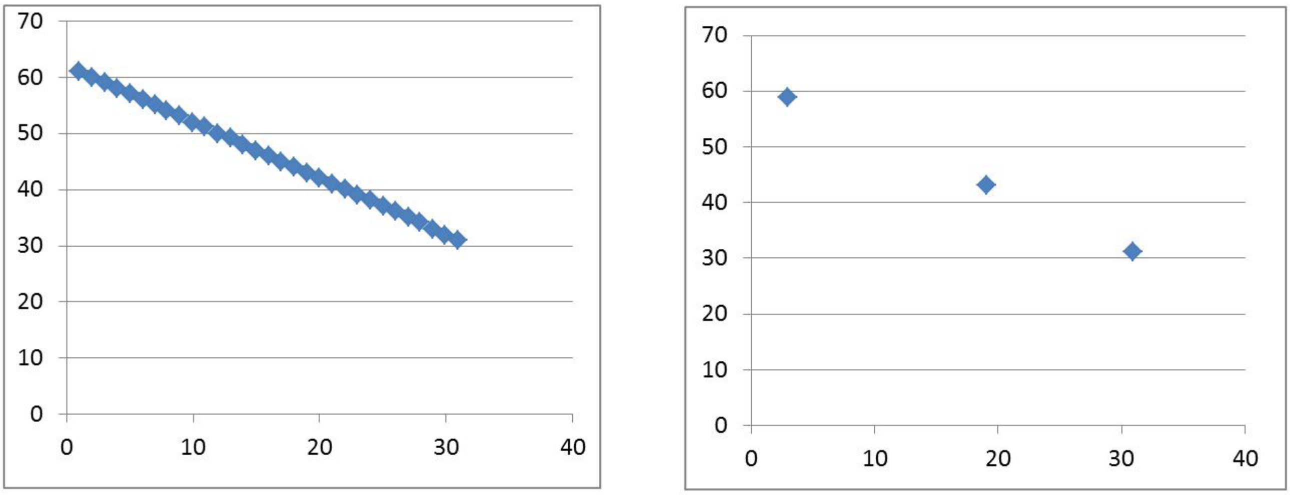


Figure 3: These graphs demonstrate linear relation for G.C.P.P (b)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
61	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32

p	3	19	31
q	59	43	31

Table 5: Goldbach Conjecture and Conjugates Pair of Primes for N=62



(a) (b)

Figure 4: Graphs (a) and (b) demonstrate the linearity between Goldbach Conjugate Pair of Primes

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